## Montague <br> Township School District

K-6

Mathematics Curriculum

The Montague Township School Districts K-8 Math Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, utilizing technology, conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to communicate mathematically and think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else-without having to commit the information to memory as a separate fact. It is the connections, the reasoned, logical connections and common vocabulary that make mathematics manageable. As a result, implementation of New Jersey Student Learning Standards places a greater emphasis on problem solving, reasoning, representation, connections, and communication.

## Format

- Each grade-level contains:
- a list of grade level critical areas
- descriptions of specific grade level mathematical practices
- grade level vocabulary.
- Framework unit overview
- An explanation of specific standards, with detailed descriptions, suggested Mathematical Practices, and Critical Knowledge and Skills
- Curriculum implementation support


## Frameworks

The New Jersey Department of Education's Division of Teaching and Learning has developed new curricular frameworks for Mathematics for kindergarten through grade twelve. The frameworks are aligned with the newly adopted New Jersey Student Learning Standards for English language arts and mathematics and will replace the model curricula for those subjects.

The purpose of the frameworks is to provide educators with a tool to guide conversations around curriculum and instruction that should be taking place in schools/districts around the state. The frameworks focus on the standards and skills in order to provide a logical sequence of instruction with the goal of mastering the standards at each grade level. It is not intended to remove teacher autonomy; rather, it is the hope that the frameworks will provide a logical, rigorous, yet flexible path through which to meet instructional goals for all students. Units within the curricular framework for mathematics are designed to be taught in the order in which they are presented in kindergarten through grade twelve. There is a logical and developmentally-appropriate progression of standards, with strong consideration given to Major, Supporting, and Additional content standards presented because most concepts build upon each other. Within the units, there is flexibility of what order to present the standards. Major, Supporting and Additional Content standards are color coded for the districts to understand where to prioritize. The intent of the standards is to integrate the Major, Supporting and Additional standards. The order in which the standards and mathematical practices are clustered within the units is a suggested integration.

## Grade Level Targets and Priority Concepts

Major, Supporting and Additional clusters of mathematics content standards are based on the New Jersey Student Learning Standards. Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, time needed to master or model, and/or their importance to future grade levels. The standards in the framework are color coded as Major (green), Supporting (blue) and Additional (yellow). Suggested Mathematical Practice Standards are listed in each unit to be imbedded regularly in daily math instruction. It is important to note that the Major standards (green) are purposefully placed in tested grades for ensuring time for formal instruction in those standards.

## Vocabulary: Building the Language of Mathematics for Students

Mathematically proficient students communicate precisely by engaging in discussions about their reasoning using appropriate mathematical language. The terms students should learn to use at each grade level with increasing precision are included in this document.

Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematic productively. Communication plays an important role in helping children construct links between their formal, intuitive notions and the abstract language and symbolism of mathematics; it also plays a key role in helping children make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas.
Curriculum and Evaluation Standards for School Mathematics, the National Council of Teachers of Mathematics (p. 26)

Mathematical vocabulary however should not be taught in isolation where it is meaningless and just becomes memorization. We know from research that meaningless memorization is not retained nor will it help build the deep understanding of the mathematical content. The students must be provided adequate opportunities to develop vocabulary in meaningful ways such as mathematical explorations and experiences. Students should be immersed into the mathematical language as they experience rich high-level tasks. As student communicate their thoughts, ideas, and justify the reasonableness of their solutions the mathematical language will begin to evolve. Student will then build the depth of understanding needed with mathematical vocabulary and content to empower them to be successful in mathematics.

## Fluency

The New Jersey Student Learning Standards for Mathematics call for students to obtain and demonstrate not only conceptual understanding and problem solving, but also procedural skill and fluency. Documents released by Student Achievement Partners go on to describe procedural skill and fluency as speed and accuracy in calculation that enable students to apply mental resources to more complex concepts and processes. Standards in grades K-6 use the word fluent, or fluently, to explicitly call for students to achieve quickness and accuracy in calculations. In later grades the standards themselves do not use the word fluent or fluently. By middle school, fluency is less about calculation, and more about ease of manipulation of expressions, equations, notations, etc. Thus, fluency exercises are applied toward any skill needed to manipulate expressions and equations with ease. For example, in Grade 8 students should work towards quickly and accurately solving general, one-variable linear equations.

When students are able to demonstrate fluency, they are accurate, efficient, and flexible. Students must have
efficient strategies in order to know sums from memory.

Research indicates that teachers' can best support students' memorization of sums and differences through varied experiences such as, making 10, breaking numbers apart and working on mental strategies. These strategies replace the use of repetitive timed tests in which students try to memorize operations as if there were not any relationships among the various facts. When teachers teach facts for automaticity, rather than memorization, they encourage students to THINK about the relationships among the facts. (Fostnot \& Dolk, 2001)

It is no accident that the standard says "know from memory" rather than "memorize". The first describes an outcome, whereas the second might be seen as describing a method of achieving that outcome. So, no, the standards are not dictating timed tests. (McCallum, 2011)

The beginning units in kindergarten through grade two are designed with more time spent on foundational mathematical concepts needed for future units to build towards fluency in mathematics. For example, kindergarten Unit 1 begins with "Number Names and Counting Sequence" which is necessary to teach prior to "Foundations with Models for Addition and Subtraction" in Unit 2. Other units in grades three through five also follow the same logical progression of standards ensuring enough time for formal instruction with the Major Standards while still embedding Supporting and Additional Content Standards as well as Mathematical Practice Standards. It is also important to note that the fluency requirement for kindergarten through grade five is critical for students to master.

## Grade Level Fluencies K-8:

Kindergarten: Add/subtract within 5
Grade 1: Add/subtract within 10
Grade 2: 2 Add/subtract within 201 Add/subtract within 100 (pencil and paper)
Grade 3: 3 Multiply/divide within 1002 Add/subtract within 1000
Grade 4: 4 Add/subtract within $1,000,000$
Grade 5: Multi-digit multiplication
Grade 6: Multi-digit division Multi-digit decimal operations
Grade 7: Solve $\mathrm{px}+\mathrm{q}=\mathrm{r}, \mathrm{p}(\mathrm{x}+\mathrm{q})=\mathrm{r}$
Grade 8: Solve simple 22 systems by inspection K Add/Subtract within 5

## Expectations or Examples of Culminating Standards for Grades 6-8

## Grade 6

6.NS. 2 Students fluently divide multi-digit numbers using the standard algorithm. This is the culminating standard for several years' worth of work with division of whole numbers.
6.NS. 3 Students fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. This is the culminating standard for several years' worth of work relating to the domains of Number and Operations in Base Ten, Operations and Algebraic Thinking, and Number and OperationsFractions.
6.NS. 1 Students interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions. This completes the extension of operations to fractions.

## Grade 7

7.EE. 3 Students solve multistep problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. This work is the culmination of many progressions of learning in arithmetic, problem solving and mathematical practices.
7.EE. 4 In solving word problems leading to one-variable equations of the form $\rangle\rangle\rangle\rangle+\rangle\rangle=\langle \rangle$ and $\geqslant\rangle(\geqslant\rangle+\rangle)=\geqslant\rangle$, students solve the equations fluently. This will require fluency with rational number arithmetic (7.NS.1-3), as well as fluency to some extent with applying properties operations to rewrite linear expressions with rational coefficients (7.EE.1).
7.NS.1-2 Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system will continue to develop in grade 8 , expanding to become the real numbers by the introduction of irrational numbers, and will develop further in high school, expanding to become the complex numbers with the introduction of imaginary numbers. Because there are no specific standards for rational number arithmetic in later grades and because so much other work in Grade 7 depends on rational number arithmetic (see below), fluency with rational number arithmetic should be the goal in Grade 7.

## Grade 8

8.EE. 7 Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in Grade 8 with the solution of general onevariable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Coefficients and constants in these equations may be any rational numbers.
8.G.9 When students learn to solve problems involving volumes of cones, cylinders, and spheres-together with their previous Grade 7 work in angle measure, area, surface area and volume (7.G.4-6)-they will have acquired a well-developed set of geometric measurement skills. These skills, along with proportional reasoning (7.RP) and multistep numerical problem solving (7.EE.3), can be combined and used in flexible ways as part of modeling during high school-not to mention after high school for college and careers.

## Problem Solving

Situation Types for Operations in Word Problems Addition and subtraction word problems are the work of the entire K-2 grade band, not the subject of a single lesson or unit. The strong focus of the Standards is intended to give teachers and students the time they need. The tables below show the 15 'situation types' or the categories of word problems required by the Standards in which the given numbers and the unknowns are in a variety of configurations. These are excerpted from the progression document, K, Counting and Cardinality; K-5, Operations and Algebraic Thinking. The first table shows distinct types of addition and subtraction situations: add to, take from, put together/take apart, and compare. Practice with word problems begins simply in kindergarten, but students must leave grade 2 with a strong command of all fifteen situation types within 100. Students develop meanings for addition and subtraction as they encounter basic word problems in kindergarten for numbers within 10. This work extends as students are introduced to more sophisticated problem types in grade 1 and begin to solve for numbers within 20. By the end of grade 2, students build on their previous practice and are solving for all fifteen situation types for numbers within 100. These tables are excerpted from the progressions document. The second table shows distinct types of multiplication and division situations: equal groups of objects, arrays of objects, and compare. Students begin word problems with these situations in
grade 3 and continues through grade 5. Although students initially learn and solve the situation types with whole numbers, this work progresses into word problems involving all rational numbers as well.

## K-2 Common Addition and Subtraction Situations

|  | RESULT UNKNOWN | CHANGE UNKNOWN | START UNKNOWN |
| :---: | :---: | :---: | :---: |
| ADD TO | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2+3=$ ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two $2+$ ? $=5$ | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? $+3=5$ |
| TAKE FROM | Five apples were on the table. I ate two apples. How many apples are on the table now?5-2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?5 - ? $=3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? $-2=3$ |
|  | TOTAL UNKNOWN | ADDEND UNKNOWN | BOTH ADDENDS UNKNOWN |
| PUT <br> TOGETHER <br> / TAKE <br> APART | Three red apples and two green apples are on the table. How many apples are on the table? $3+2=$ ? | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3+?=5,5-3$ $=$ ? | Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5=$ $\begin{aligned} & 0+5,5+05=1+4,5=4+15=2 \\ & +3,5=3+2 \end{aligned}$ |
| COMPARE | DIFFERENCE UNKNOWN | BIGGER UNKNOWN | SMALLER UNKNOWN |
|  | ("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? $2+$ ? $=5,5-2$ $=$ ? | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2+3=?, 3+2=$ ? | (Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): <br> Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5-3=$ ?, $?+3=5$ |

1 Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33). 2 These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the $=$ sign does not always mean, makes or results in but always does mean is the same number as. 3 Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10 .
4 For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult

## Grade 3-5 Common Multiplication and Division Situations

|  | UNKNOWN PRODUCT | GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION) | NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION) |
| :---: | :---: | :---: | :---: |
|  | $3 \times 6=$ ? | $3 \times ?=18$, and $18 \div 3=$ ? | ? x $6=18$, and $18 \div 6=$ ? |
| EQUAL GROUPS | There are 3 bags with 6 plums in each bag. How many plums are there in all? <br> Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| ARRAYS AREA | There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| COMPARE | A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $\$ 18$ and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $\$ 18$ and a blue hat costs $\$ 6$. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| GENERAL | $\mathrm{a} \times \mathrm{b}=$ ? | $\mathrm{ax} ?=\mathrm{p}$ and $\mathrm{p} \div \mathrm{a}=$ ? | $? \mathrm{xb}=\mathrm{p}$, and $\mathrm{p} \div \mathrm{b}=$ ? |

1 The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable. 2 Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
3 The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

## 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions
and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5+7 \times 3$, in preparation for learning about the distributive property. In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3 , middle school students might abstract the equation $(y-2) /(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$, and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical PRactice to the Standard for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

## Unit Design

Each curriculum unit is designed within the Understanding by Design (UbD) framework. Stage One focuses on the 'Desired Results' or, the 'what', of the curriculum. This stage includes New Jersey Student Learning Standards for Mathematics, Technology, and 21 st Century Life and Career. In addition, Enduring Understandings, Essential Questions, Knowledge and Skills are specifically outlined. Stage One indicates what students need to understand, what they will keep considering, what they will know and what they will be able to do. The items in Stage One of each unit provide the framework that teachers must follow in order to ensure that the New Jersey Student Learning Standards and curriculum objectives are met.

In Stage Two (Assessment Evidence), students will display that they have achieved the goals of Stage One. This section outlines specific assessment and performance tasks that students will engage in to display their level of understanding of unit content. Assessments and performance tasks are written specific to the content taught in each unit. These assessments are varied, including but not limited to, formative assessments, summative assessments, alternative assessments and benchmark assessments.
*Additional information on Assessment can be found in Appendix B.

In Stage Three of the framework, the Learning Plan is outlined with key learning events and instruction. This is considered to be the 'how' of the curriculum. In this section, exemplary learning activities, integrated accommodations, integrated modifications, interdisciplinary connections, technology integration, 21 st century life and career integration activities are suggested. Depending on the individual needs of the district and the students in each classroom, teachers are expected to differentiate the Stage Three components as needed. Differentiation of content, process and/or product will be necessary depending upon the strengths and needs of the students in the classroom.

## Meeting the Needs of Diverse Learners through Differentiation

Classrooms are dynamic centers that include students of all backgrounds, ability levels, and interests. In order to meet the specific needs and capitalize on the specific strengths of individual students, differentiation is key. Effective instruction must include a teacher's commitment to a high level of differentiation. Modifications are designed to change the learning goal and/or objective. Accommodations change the way a student receives information or is tested without changing the learning goal. Integrated modifications, accommodations and differentiation strategies have been built into each unit, at every grade level,
throughout this curriculum. These are specific to the content studied in each unit and target the following student populations:

1. Special Education Students
2. English Language Learners
3. Students At Risk of School Failure
4. Gifted and Talented Students
5. Students with 504 Plans

In order to fully meet the needs of students, the implementation of Response to Intervention is also necessary. In 2016, the New Jersey Department of Education (NJDOE), in collaboration with educators, higher education representatives and parents, has developed a set of resources for districts to facilitate implementation of RtI known as "New Jersey Tiered System of Supports (NJTSS)". NJTSS includes the three-tiered approach to instruction, assessment and intervention found in many models of response to intervention, along with three foundational components: effective district and school leadership, positive school culture and climate, and family and community engagement. Together, these components create an efficient and effective mechanism for schools to improve achievement for all students. NJTSS builds on effective practices and initiatives already in place in schools, and maximizes the efficient use of resources to improve
support for all classroom teachers and target interventions to students based on their needs. An RtI program consistent with section 100.2 (ii) of the Regulations of the Commissioner must include the following minimum components: $\bullet$ Appropriate instruction delivered to all students in the general education class by qualified personnel. Appropriate instruction in reading means scientific research-based reading programs that include explicit and systematic instruction in phonemic awareness, phonics, vocabulary development, reading fluency (including oral reading skills) and reading comprehension strategies.

- Screenings applied to all students in the class to identify those students who are not making academic progress at expected rates.
- Instruction matched to student need with increasingly intensive levels of targeted intervention and instruction for students who do not make satisfactory progress in their levels of performance and/or in their rate of learning to meet age or grade level standards.
- Repeated assessments of student achievement which should include curriculum based measures to determine if interventions are resulting in student progress toward age or grade level standards.
- The application of information about the student's response to intervention to make educational decisions about changes in goals, instruction and/or services and the decision to make a referral for special education programs and/or services.
- Written notification to the parents when the student requires an intervention beyond that provided to all students in the general education classroom that provides information about the:
- amount and nature of student performance data that will be collected and the general education services that will be provided
$\circ$ strategies for increasing the student's rate of learning
- parents' right to request an evaluation for special education programs and/or services.

1. Requires each school district to establish a plan and policies for implementing school-wide approaches and pre-referral interventions in order to remediate a student's performance prior to referral for special education, which may include the RtI process as part of a district's school-wide approach. The school district must select and define the specific structure and components of its RtI program, including, but not limited to:
a. criteria for determining the levels of intervention to be provided to students
b. types of interventions
c. amount and nature of student performance data to be collected
d. manner and frequency of progress monitoring
2. Requires each school district implementing a RtI program to take appropriate steps to ensure that staff have the knowledge and skills necessary to implement a RtI program and that such program is implemented in a way that is consistent with the specific structure and components of the model.
*See Appendix A for comprehensive accommodations and modifications for the above student populations and the

## 21st Century Life and Career

One of the goals of the Montague Township School is to prepare our students for success as contributing citizens in the 21 st Century. New Jersey Student Learning Standards for 21st Century Life and Career are integrated in each unit of study at every grade level. In addition, suggested learning activities are outlined to meet the standards selected for the unit.

## Interdisciplinary Connections

This Mathematics curriculum supports additional core curriculum content areas. Interdisciplinary learning is one of many ways to learn over the course of a curriculum. Montague chooses interdisciplinary learning to deliver the Mathematics content. The Region is especially committed to the integration of other subject areas into the Mathematics curriculum as children are able to technology, science and engineering concepts. This method brings students to a new awareness of the meaningful connections that exist among the disciplines and allows them to synthesize information. Each unit of study includes suggested learning activities to integrate other disciplines.

## Technology Integration

Technology plays an integral part in the teaching and learning process throughout the Montague School. Students utilize technology to access the curriculum, learn new content and apply their knowledge in a variety of ways. New Jersey Student Learning Standards for Technology are integrated in each unit of study at every grade level. In addition, suggested learning activities are outlined to meet the standards selected for the unit. Montague has a wide range of media and technology available for staff and student use. Each district has made significant strides toward training their staff in terms of integrating technology into all curriculum areas. In addition, we have plans to continue to expand the technology we have and to extend the training offerings available to staff. Technology plays an important role in the implementation of the Social Studies curriculum. Within the limits of available equipment and materials, teachers in the various districts will make regular, appropriate use of the available technology and media.

